

Test 1A, Math of Democracy

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18 min ✓

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(signature)

DIRECTIONS

1. Don't panic.
2. I don't hate you (but somebody does).
3. **Show/explain all of your work.** A correct answer with insufficient work will lose points.
4. Read each question carefully, and make sure you answer the question that is asked. If the question asks for an explanation, make sure you give one.
5. Clearly indicate your answer.
6. Calculators are allowed on this test, but any other technology (cell phones, computers, etc) is NOT allowed without prior authorization.
7. Make sure you sign the pledge.
8. Number of questions = 4. Total Points = 20.

1. (6 points) The "Later No Harm" criterion is satisfied if, in any election for a particular voting method, a voter (or group of voters) giving an additional ranking or positive rating to a less-preferred candidate can not cause a more-preferred candidate to lose. For example, the Bucklin method *fails* Later No Harm, which you can see from this example:

Number of voters	8	2	4	6	3
1st choice	A	B	C	D	D
2nd choice	C	A	D	B	C
3rd choice	D	D	B	A	B
4th choice	B	C	A	C	A

C wins the Bucklin method because no candidate has a majority of first-place, but when you include 2nd place votes, C has 15 ($8 + 4 + 3$) to D 's 13 ($4 + 6 + 3$) votes. However, if the the 3 $DCBA$ voters had just voted D , the preference schedule would be:

Number of voters	8	2	4	6	3
1st choice	A	B	C	D	D
2nd choice	C	A	D	B	
3rd choice	D	D	B	A	
4th choice	B	C	A	C	

Now D would win, because they have 13 votes in the second round but C only has 12. This is a violation, because for the $DCBA$ voters, adding C as their 2nd-place rank caused their 1st place candidate D to lose the election.

Questions:

- The Top-two runoff and Instant Runoff methods both *satisfy* Later No Harm. Choose one of those methods and explain why it satisfies Later No Harm.
- Approval and Range voting both *fail* Later No Harm. Choose one of those methods and explain why it violates Later No Harm.

You can use the back side of this page if you need more space.

(a) For Instant runoff, as long as your ^{preferred (top-ranked)} candidate is still in the race, the rankings below are not counted at all. Thus adding more lower-placed rankings cannot ~~effect~~ ^{hurt} the outcome ~~of~~ of your preferred candidate, because those lower rankings do not even factor in. Example: Suppose candidate A wins an IRV election. Any rankings below A in any column would never get counted. Thus having them (or not) does not change the result.

Same argument holds for Top-two runoff

if do not-so-good example and little reasoning for why in general

(2/3) x

2. (5 points) Below are listed 4 pairs of voting methods and fairness criteria.

- (a) Choose one of the pairs below and explain why that particular voting method *satisfies* that particular criteria.
- (b) Choose one of the pairs below and explain why that particular voting method *violates* that particular criteria.

If you want to do more than one in a category, that is fine. I will score both answers and give you the higher of the two scores.

- Range voting and No-show criterion S
- Instant Runoff voting and Independence of Irrelevant Alternatives criterion F
- Borda Count and Independence of Irrelevant Alternatives criterion F
- Approval voting and Monotonicity criterion S

2.5 each
#1 for correct, 1.5 for explanation.

IRV fails IIA b/c of the following example:

<u>35</u>	<u>33</u>	<u>32</u>
A	B	C
B	C	A
C	A	B

C drops first, A wins.

But, if B were not in the race, C would win with a majority of 1st place votes (65). So IRV fails IIA.

Max if
2/2.5 if
No concrete
example
attempt.

Approval Satisfies Monotonicity b/c "raising a ranking" in Approval voting just means giving some candidate an "Approve" instead of "disapprove". If Candidate B was winning an Approval vote, and you change some ballots to give them more approvals, ~~the~~ B would still win, just by a larger margin.

(b) Range voting fails Late No Harm. For example,
 Consider the following Range vote, with only 3 voters

2.5/3
 if give good
 explanation but
 no example.

Voter 1:	Voter 2:	Voter 3:
A → 10	B → 10	C → 10
B → 9	A → 3	A → 1
C → 2	C → 1	B → 1

B wins with 20 points to A's 14 points. If
 A Voter 1 had NOT given points to B (Who they
 ranked lower), then their most-preferred candidate
 A would have won with 14 points to B's 11
 and C's 12 points.

Similar argument holds for Approval. Suppose
 If everyone Approves of their top two candidates,
 A wins. If the 10 BAC only
 approve of B, then B wins. So by
 "approving" of their less-favored candidate, those
 voters made their more favored candidate
 lose.

Approval	11	10	5
→	A	B	C
	B	A	A
	C	C	B

Range voting Satisfies No-Show. Assuming that each voter gives higher scores to their more-preferred candidates, not showing up only loses more points for your favorite candidates. For example, suppose A ~~was~~ lost a range vote election, and B won (win 100 points). If a voter who ranked ~~B~~ ^A higher (say $A=10, B=9$) ^(w 99 pts) does not show up, then A loses more points (Now A would have 89 points and B would have 91, so B still wins). Thus there is no way that losing candidate could somehow win and cause an anomaly.

Borda Count fails IIA. Consider this election where B, B_1 are clones.

~~100~~
100

$\frac{60}{A}$	$\frac{40}{B}$
B	B_1
B_1	A

Borda (2-1-0): $A = 120$
 $B = 60 + 80 = 140$
 $B_1 = 40$

B wins!

If B_1 was not in the race, though, you have

$\frac{60}{A}$	$\frac{40}{B}$
B	A

and A wins the Borda Count

3. (5 points) Consider the following preference schedule

Number of voters	26	23	3	16	16	16
1st choice	A	A	C	C	B	D
2nd choice	B	D	A	B	D	B
3rd choice	C	C	B	D	A	C
4th choice	D	B	D	A	C	A

(a) Calculate the winner of the election using the Coombs method. Show/explain your work.

✓ No majority. A drops w/ 32 last-place voters
 Now B=42, C=19, D=39. Last place votes: C has 32,
 D=45, B=23, so D drops and 16 votes go to B,
B wins w/ 58 votes.

(b) Now suppose that there was a slight change and instead, the following election happened:

Number of voters	26	23	16	16	16
1st choice	A	A	C	B	D
2nd choice	B	D	B	D	B
3rd choice	C	C	D	A	C
4th choice	D	B	A	C	A

Who would be the winner of the Coombs election now? Show/explain your work.

✓ A wins with a majority in Round 1.

(c) What issue do the examples above illustrate about the Coombs method? (Hint: what is different between the first and second examples, and how does that affect the winner of the election?)

✓✓✓ 3 CABD voters did not show in election 2. When they did show up (election 1) their 3rd-place candidate wins. When they don't show up, their 2nd place candidate wins. So they are better off not voting, which is a ~~violation of~~ no-show ^{anomaly}. This shows that Coombs violates the No-show criterion.

at least -1 if don't mention No-show.

3/5 if mess up #2 but state that it is a no-show

4. (4 points) Suppose the High Point University student body president decides that our current student government election method is bad, and they want to come up with a ranked-choice method to replace it with—which of the methods we have studied (or one you would create yourself) would you recommend? Why would you recommend it? Make sure to explain the benefits and drawbacks of your choice, and to compare to other methods to illustrate why yours is better.

I would recommend Black's method, where you look for a Condorcet winner. If there is a Cond. winner, that person wins the election. If not, you do a Borda count. This method satisfies the majority criterion (as opposed to Borda), but if you have no Cond. winner (which could also cause a tie in a method like pairwise) you still have a backup method to make a winner to the election. Black's method also takes into consideration all rankings ~~at~~ from the beginning (as opposed to Plurality, Top-two, or IRV), which ~~clearly~~ indicates it is taking in more information and is thus more "democratic" from that perspective. It would also be an easy transition for a current plurality method, since the majority winner would still win the election.

(Note: Many answers)

-0.5 if no (or few) comparisons

-0.5 if no mention of any fairness criteria.

Extra Credit (up to 1.5 points) Choose 0.5 or 1.5 points to have as extra credit. If you put 0.5, you are guaranteed to get 0.5 extra credit points. If you choose 1.5, you are taking a chance: If three or more students in the class (including yourself) put 1.5, then everyone who puts 1.5 gets no points. If only one or two students choose 1.5, then they get the 1.5 points.

Q1:

<u>0.5</u>	<u>1.5</u>	<u>None</u>



Q2:

<u>0.5</u>	<u>1.5</u>
b	

Description of Methods:

- Plurality: Candidate with the highest number of first-place votes wins the election
- Top-two runoff: Eliminate all candidates *except* the two candidates with the most first-place votes. Whoever wins a head-to-head between the top two candidates is the winner of the election.
- Pairwise Comparison: Each candidate does a head-to-head contest against each other candidate. Winning a head-to-head gives a candidate 1 point. Whoever has the most points wins the election.
- Borda Count: Each candidate receives points (in descending order) based on their ranking, for example: 2 points for a first-place vote, 1 point for a second-place vote, and no points for a 3rd-place vote. All the points are totaled and whoever has the most points wins the election.
- Instant Runoff: If no candidate has a majority of first-place votes, the candidate with the fewest first-place votes is eliminated and then there is another round of voting. Rounds continue with candidates being eliminated until one candidate has a majority of the first-place votes.
- Coombs: If no candidate has a majority of first-place votes, the candidate with the most last-place votes is eliminated and then there is another round of voting. Rounds continue with candidates being eliminated until one candidate has a majority of the first-place votes.
- Sequential Pairwise: Candidates are paired up, in a particular order called an *agenda*, to compete in head-to-head contests. If a candidate wins a head-to-head, they advance to the next round and compete against the next candidate in the agenda. The winner after all of the rounds is the winner of the election.
- Schulze: A graph/network diagram is made showing the candidates, who they beat in head-to-head contests, and by how much. Then each pair of candidates is compared, and the 'winner' of each comparison is the candidate with the strongest winning path over the other candidate. The candidate who has a stronger winning path over each of the other candidates is the winner.
- Range: Each voter gives each candidate a score from 0 to 10. Points are tallied and the candidate with the highest score wins.
- Approval: Each voter gives each candidate an approval or not. Approvals are tallied and the candidate with the highest number of approvals wins.
- Bucklin: All first-place votes are counted. If a candidate has a majority of the vote, that candidate wins. If no candidate has a majority, then 2nd-place votes are added to first-place votes. If one candidate has a majority, that candidate wins. If multiple candidates have a majority, then whichever candidate has the most votes wins the election. If no candidate has a majority, then third-place votes are included, and the process continues.

Description of Fairness Criteria:

- Majority Criterion: If a candidate wins a majority of the first-place votes, then that candidate should win the election.
- (upward) Monotonicity Criterion: If you raise a candidate's ranking on some ballots, that should never hurt the candidate (in particular, it should never make the candidate LOSE an election that they otherwise would have won).
- Independence of Irrelevant Alternatives Criterion: Removing a losing candidate from the ballot should never change the winner of an election.
- No-show Criterion: It should never benefit a voter (or group of voters) to NOT cast their ballot.
- Clone-proof Criterion: It should never hurt a candidate to have other *similar* candidates also in the race with them (that is, you should not allow similar candidates to split their vote and end up hurting their chance of winning). Clones of a candidate should also not *help* that candidate.
- Condorcet Winner Criterion: If there is a Condorcet *Winner* in a preference schedule (that is, a candidate who beats every other candidate in a head-to-head) then that candidate should win the election.
- Condorcet Loser Criterion: If there is a Condorcet *Loser* in a preference schedule (that is, a candidate who loses to every other candidate in a head-to-head) then that candidate should NOT win the election.
- Later No Harm Criterion: A higher-ranked candidate should not be harmed by a voter (or group of voters) giving an additional ranking or positive rating to a less-preferred candidate.

